Let y = total cost and let x = the variable of design. The situation of cost variation just described may be expressed by the equation $y = ax + \frac{b}{x} + c$

Taking the first derivative, we find

$$\frac{dy}{dx} = a - \frac{b}{x^2}$$

Equating this to zero, and solving for x,

$$x = \sqrt{\frac{b}{a}}$$

This is the value of the design variable that makes cost a minimum.

When $x = \sqrt{\frac{b}{a}}$, the directly varying costs equal the inversely varying costs. This fact is illustrated in Figure 10–1 and may be demonstrated as

$$ax = a\sqrt{\frac{b}{a}} = \sqrt{ab}; \frac{b}{x} = \frac{b}{\sqrt{\frac{b}{a}}} = \sqrt{ab}$$

The formula $x = \sqrt{\frac{b}{a}}$ can be applied to different kinds of problems, but it